Design of Code Division Multiple Access Filters Using Global Optimization Techniques

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Abstract

A semi-deterministic global optimization method based on the search of a suitable initial condition for a given optimization algorithm is presented. This method is applied to the design of code division multiple access filters used in data transmission and is compared, in term of complexity and final design, with a genetic algorithm.

Keywords: *Global optimization; Genetic algorithms; Descent algorithms; Optical fibers design; Code division multiple access.*

1 Introduction

The use of optical fibers offering bandwidth of several TeraHertz per telecommunication window in the telecommunication sector has known important developments in the last decade (Skaar J. et al., 1998; Yang C.C., 2008) with applications in television by cable or cell phones (Takeshi F. et al., 2007; Yang H.K. et al., 1999). However, to be fully efficient, the fibers should allow multiple access. This means allowing various persons (called **users**) to send and receive messages in the fiber at the same time (See Figure 1).

There exist three main schemes of multiple access: Time Division Multiple Access (TDMA) (Han K.E. et al., 2006) which allows to consider a great number of users but requires fast synchronization, Wavelength-Division Multiplexing (WDM) (Bock C. et al., 2005) which sometimes requires precise adjustments, and Code Division Multiple Access (**CDMA**) (Moreau Y. et al., 2000) which primarily allows a great flexibility in multiple accesses. This last technique presents various advantages such as high fidelity and high resistance to signal perturbations, secured communications and low power consumption (Viterbi A.J., 1995).

In this paper, we focus on the design of a particular optical filter based on the **CDMA** technique. Currently, there is an important demand of optimization methods for the design of such filters. However, those methods need to perform global optimization as it has been observed that the functionals involved in the design have multiple minima (Skaar J. et al., 1998).

Genetic algorithms appear then as a natural choice (Forrest S., 1993; Goldberg D., 1989). But, we would like to see if the target cannot be achieved with a lower computational effort taking advantage of low-complexity features of a semi-deterministic global optimization method (Ivorra , 2006; Ivorra B. et al., Submitted, 2007). In order to check the efficiency of the method, we compare the results with those obtained with a genetic algorithm.

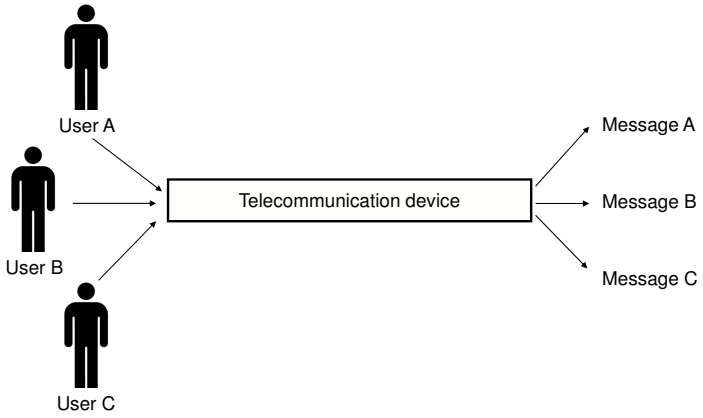

Figure 1: Multiple access technique principle: Users A,B and C send at the same time a message in a communication device. The multiple access technique allows to properly deliver each message.

The paper is organized as follows. Section 2 briefly describes the CDMA filter, its mathematical modeling and the design problem. Section 3 presents the optimization methods. Finally, in Section 4 we show a particular design problem and compare the obtained results in term of numerical and implementation performances.

2 CDMA filter design problem

2.1 CDMA principle

In the basic technique of CDMA, the bits '1' or '0' of a binary message, send by an user, are replaced at the level of the transmitter by codes attributed to this user.

We can consider binary codes of length $N_{\text{code}} \in \mathbb{N}$. The code for the bit '1' of a particular user 'A' is denoted by $c_1^A \in \{0,1\}^{N_{\text{code}}}$ and its complement, denoted by $c_0^A = -(c_1^A - 1)$, is used for '0' (for instance, if $N_{\text{code}} = 8$, the possible codes are c_1^A ='10110011' and c_0^A ='01001100'). During this work we will only focus on this binary coding technique.

Zaccarin and Kavehrad (Kavehrad M. et al., 1995) and later Lam (Lam C. et al., 1999) suggested to use spectra compound by a set of wavelengths $\Lambda = (\lambda_i)_{i=0}^{N_{\text{code}}}$ in order to represent those codes (i.e. the reflectivity of λ_i is equal to $c(i)$ for $i = 1, ..., N_{\text{code}}$, where c is the considered code, and zero elsewhere). One way to generate such a spectrum is to consider Sampled Fiber Bragg Gratings (**SFBG**).

SFBGs are optical fibers based on a periodic perturbation of their refractive effective index, obtained by exposing the optical fiber to UV radiations (Erdogan T., 1997). The objective of SFBGs are to reflect predetermined wavelengths and to let other wavelengths pass (Chow J. et al., 1996; Wei D. et al., 2000). They can be easily hybridized with other optical devices (Helmers H. et al., 2002) such as optical isolators presented in Section 2.2.

2.2 CDMA device to be designed

We propose to design a part of a transmitter based on the CDMA codification presented in Section 2.1. More precisely, we consider the **code separator** of a particular user 'A' depicted in Figure 2. The objective of this code separator is to separate the spectra corresponding to c_1^A and c_0^A. It is formed by three components:

- An optical isolator where forward signals are directed to a CDMA filter and backward ones to a classical optical fiber.
- A **CDMA filter** compound by a SFBG that reflects the spectrum corresponding to c_1^A.
- A classical optical fiber that let a signal pass.

Here, we focus on the design of the CDMA filter. To do so, we first present a mathematical model used to compute the reflected spectrum of SFBGs.

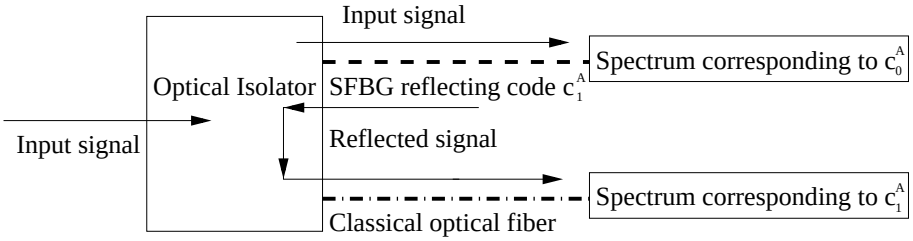

Figure 2: Code separator.

2.3 SFBG reflected spectrum evaluation model

We assume that in a SFBG, for any wavelength λ (μm) in a considered transmission band $[\lambda_{\min}, \lambda_{\max}]$, there exist only two counter-propagating guided modes: a transmitted and a reflected mode, of respective amplitudes $T(.,\lambda)$ and $R(.,\lambda)$. Due to the presence of a dielectric perturbation, those modes are coupled through the following equations (Skaar J. et al., 1998):

$$\begin{cases} \dfrac{\mathrm{d}B}{\mathrm{d}z}(z,\lambda) = i\hat{\sigma}(z,\lambda)B(z,\lambda) + i\kappa(z,\lambda)C(z,\lambda) & ,\lambda \in [\lambda_{\min},\lambda_{\max}], z \in [0,L], \\ \dfrac{\mathrm{d}C}{\mathrm{d}z}(z,\lambda) = -i\hat{\sigma}(z,\lambda)C(z,\lambda) - i\overline{\kappa}(z,\lambda)B(z,\lambda) & ,\lambda \in [\lambda_{\min},\lambda_{\max}], z \in [0,L], \\ B(0,\lambda) = 1 & ,\lambda \in [\lambda_{\min},\lambda_{\max}], \\ C(L,\lambda) = 0 & ,\lambda \in [\lambda_{\min},\lambda_{\max}], \end{cases} \tag{1}$$

where $B(z,\lambda) = T(z,\lambda)\exp(i\zeta(\lambda)z - \frac{\phi(z)}{2})$, $C(z,\lambda) = R(z,\lambda)\exp(-i\zeta(\lambda)z + \frac{\phi(z)}{2})$, L is the fiber length in millimeters (mm), $\hat{\sigma}(z,\lambda) = \zeta(\lambda) + \beta(\lambda)\frac{\overline{\delta n_{\text{eff}}(z)}}{n_{\text{eff}}} - \frac{1}{2}\frac{d\phi}{dz}(z)$ is the demi-coupling coefficient, $\kappa(z,\lambda) = \frac{\nu}{2}\beta(\lambda)\frac{\overline{\delta n_{\text{eff}}(z)}}{n_{\text{eff}}}$ is the associated-coupling coefficient, $\zeta(\lambda) = \beta(\lambda) - \frac{\pi}{\Theta}$ is the detuning parameter, $\beta(\lambda) = \frac{2\pi n_{\text{eff}}}{\lambda}$ is the propagation constant, n_{eff} is the unperturbed refractive effective index, Θ is the nominal grating period (μm), $\overline{\delta n_{\text{eff}}}(z)$ is the slowly varying index amplitude change over the grating (called **apodization**) produced by the UV exposure which is periodic of period P (mm), ν is the fringe visibility and $\phi(z)$ is the slowly varying index phase change (also called chirp).

In addition, in order to obtain a SBFG easy to build, we consider fibers with no chirp (i.e. $\phi(z) = 0$) and the fringe of visibility is considered to be $\nu = 1$.

System (1) can be solved using a simplified transfer matrix method (Erdogan T., 1997).

The reflected spectrum, or **power reflection function** r, of the considered SFBG is defined as:

$$\lambda \mapsto r(\lambda) = \left|\frac{C(0,\lambda)}{B(0,\lambda)}\right|^2. \tag{2}$$

2.4 Optimization problem

We consider that the code c_1^A is represented by the following set of wavelengths:

$$\Lambda_A^1 = \{\lambda_i | i = 1...N, N \leq N_{\text{code}}, \lambda_i \in \Lambda\}. \tag{3}$$

Due to the fact that the considered SFBG introduced in Section 2.3 can only generate symmetric reflected spectra (Ivorra B. et al. in IJCSE, 2006), we want to design a SFBG that reflects Λ_A^1 and also the symmetrical wavelengths of Λ_A^1 centered around a wavelength λ_c. This new set of wavelengths is denoted by Λ_{A,λ_c}^1.

This problem can be reformulated considering that each SFBG with no chirp can be characterized by its apodization $z \mapsto \overline{\delta n_{\text{eff}}}(z)$. Denoting by Ω_{apo} the search space of all admissible apodization profiles, we define a cost function h_0, to be minimized on Ω_{apo}, by:

$$h_0(x) = \int_{[\lambda_{\min}, \lambda_{\max}]} (r_x(\lambda) - r_{\text{t}}(\lambda))^2 \text{d}\lambda, \tag{4}$$

where $r_x(.)$ is the power reflection function (2) of the SFBG with an apodization associated to $x \in \Omega_{\text{apo}}$ and r_{t} is the target power reflection function given by:

$$r_t(\lambda) = \begin{cases} 1 \ if \ \lambda \in \Lambda_{A,\lambda_c}^1, \\ \\ 0 \text{ elsewhere.} \end{cases} \tag{5}$$

We must include some restrictions on Ω_{apo} in order to find a SFBG with an apodization profile with suitable characteristics for practical realization. Indeed, complex apodization profiles would require high-level and expensive mastering of the writing process. In particular, we are interested by admissible profiles which have a low number of π-phase shifts (sign changes in the profile), are smooth and have a maximum index variation $\overline{n}_{\max}$ of less than 5.10^{-4}. Thus, apodization profiles are generated by spline interpolation through a reduced number of N_S points equally distributed along the first half of the profile and completed by parity. N_S is chosen high enough to ensure enough peaks in the reflected spectra but small enough for the profile to remain admissible.

Thus, the corresponding search space of the optimization problem is a hypercube:

$$\Omega_{N_S} = [-\overline{n}_{\max}, \overline{n}_{\max}]^{N_S}, \tag{6}$$

where $\overline{n}_{\max}$ is a design constraint.

The discrete version of the cost function (4) on Ω_{N_S} is defined by:

$$h_{0,N_c}(x) = \sum_{i=1}^{N_c-1} \frac{(\lambda_{i+1} - \lambda_i)}{2} \Big[(r_x(\lambda_{i+1}) - r_{\text{t}}(\lambda_{i+1}))^2 + (r_x(\lambda_i) - r_{\text{t}}(\lambda_i))^2 \Big]. \tag{7}$$

In the above expression, the power reflection function r_x of the SFBG with an apodization associated with $x \in \Omega_{N_S}$ is evaluated on N_c wavelengths equally distributed on the transmission band $[\lambda_{\min}, \lambda_{\max}]$.

Therefore, the CDMA filter design problem can be formulated as the following optimization problem:

$$\begin{cases} \text{Find } x_m \in \Omega_{N_S} \text{ such that} \\ \\ h_{0,N_c}(x_m) = \min\limits_{x \in \Omega_{N_S}} h_{0,N_c}(x). \end{cases} \tag{8}$$

3 Global optimization methods

We consider the following minimization problem:

$$\begin{cases} \text{Find } x_m \in \Omega \text{ such that} \\ \\ h_0(x_m) = \min\limits_{x \in \Omega} h_0(x), \end{cases} \tag{9}$$

where $h_0 : \Omega \to \mathbb{R}$ is the cost function and x is the optimization parameter belonging to a convex search space $\Omega \subset \mathbb{R}^N$, with $N \in \mathbb{N}$. We assume $h_0 \in C^0(\Omega, \mathbb{R})$ is a coercive function (i.e. $\lim_{\|x\| \to +\infty} h_0(x) = +\infty$).

3.1 Genetic algorithms

Genetic algorithms, denoted as **GA**, approximate the solution of (9) through stochastic processes based on an analogy with the Darwinian evolution of species (Forrest S., 1993).

A first family, called '*population*', $X^0 = \{x_j^0 \in \Omega, j = 1, ..., N_p\}$ of $N_p \in \mathbb{N}$ possible solutions of the optimization problem, called '*individuals*', is randomly generated in Ω.

Starting from this population, we build recursively $N_g \in \mathbb{N}$ new populations $X^{i+1} = \{x_j^{i+1} \in \Omega, j = 1, ..., N_p\}$ with $i = 0, .., N_g - 1$, called '*generations*', via three main steps:

Step 1- Selection: Each individual, x_j^i, $j = 1, ..., N_p$ is ranked with respect to its cost function value $h_0(x_j^i)$ (i.e. the lower is $h_0(x_j^i)$ the higher is its ranking). Then, N_p individuals are randomly selected to become '*parents*', with a probability depending on the previous ranking (individuals with better ranking have higher chances to be selected) and with eventual repetitions.

Step 2- Crossover: This step leads to a data exchange between two parents and the apparition of two new individuals called '*children*'. We determine, with a fixed probability $p_c \in [0, 1]$, if two consecutive parents should exchange data (the created children are projected in Ω) or if they are directly copied into the new population.

Step 3- Mutation: This step leads to new parameter values for some individuals of the population. For each individual, we determine with a fixed probability $p_m \in [0, 1]$ if it is randomly perturbed (the perturbed individual is projected in Ω) or not.

With these three basic evolution processes, it is generally observed that the best obtained individual is getting closer after each generation to the optimal solution of the problem (Goldberg D., 1989).

At the end of the algorithm, after N_g iterations, the GA returns an output denoted by $A_0(X^0; N_p, N_g, p_m, p_c) = \text{argmin}\{h_0(x_j^i)/x_j^i \in X^i, i = 1, ..., N_g, j = 1, ..., N_p\}$.

These algorithms do not require sensitivity computation, perform global and multi-objective optimization and are easy to parallelize. However, their drawbacks remain their computational complexity, their slow convergence and their lack of accuracy. Since a fine convergence is difficult to achieve with GAs, it is recommended when it is possible, to complete the GA iterations by a descent method (Dumas L. et al., 2004).

A complete description of the GA considered during tis work can be found in the following literature (Ivorra , 2006; Ivorra B. et al., Accepted).

3.2 Semi-deterministic global optimization method

3.2.1 General description of the method

We consider an optimization algorithm $A_0 : V \to \Omega$, called 'core optimization algorithm', to solve (9).

We assume the existence of a suitable initial condition $v \in V$ such that the output returned by $A_0(v)$ approaches a solution of (9). In this case, solving numerically (9) with the considered core optimization algorithm can be formulated as:

$$\begin{cases} \text{Find } v \in V \text{ such that} \\ \\ v \in \text{argmin}_{w \in V} h_0(A_0(w)). \end{cases} \tag{10}$$

In order to solve (10), we propose to use a I-layer semi-deterministic algorithm $A_I : V \to V$, with $I \in \mathbb{N}$, based on line search methods (see, for instance, (Mohammadi B. et al., 2002)) called here, for the sake of simplicity, 'Semi-Deterministic Algorithm' (**SDA**) and built recursively as following:

For $i = 1, 2, ..., I$, we introduce $h_i : V \to \mathbb{R}$ by

$$h_i(v) = h_{i-1}(A_{i-1}(v)), \tag{11}$$

and we consider the problem:

$$\begin{cases} \text{Find } v \in V \text{ such that} \\ \\ v \in \text{argmin}_{w \in V} h_i(w). \end{cases} \tag{12}$$

Problem (12) is equivalent to (10) and is solved using the algorithm $A_i : V \to V$ that, for each $v_1 \in V$, returns an output given by

Step 1- Choose v_2 randomly in V.

Step 2- Find $v \in \text{argmin}_{w \in \mathcal{O}_i(v_1, v_2)} h_i(w)$, where $\mathcal{O}_i(v_1, v_2) = \{v_1 + t(v_2 - v_1), t \in \mathbb{R}\} \cap V$, using a line search method.

Step 3- Return v.

The line search minimization algorithm in Step 2 is defined by the user.

When $I > 1$, due to the fact that line search directions $\mathcal{O}_i(v_1, v_2)$ in A_i, for $i = 1, ..., I-1$, are constructed randomly, the algorithm A_I perform a multi-directional search of the solution of (10).

A detailed description of this method and various implementation schemes can be found in the following literature (Ivorra , 2006; Ivorra B. et al., Submitted, 2007; Debiane L. et al., 2006; Ivorra B. et al. in IJNME, 2006).

In Sections 3.2.2 and 3.2.3, we present two particular implementations of the SDA, considering descent and genetic algorithms as core optimization algorithms in the case where h_0 is a non negative function with zero as the minimum value.

3.2.2 SDA implementation with descent core optimization algorithms

We consider core optimization algorithms that come from the discretization of the following initial value problem (Mohammadi B. et al., 2002):

$$\begin{cases} M(x(t), t)x_t(t) = -d(x(t)), \quad t \geq 0, \\ \\ x(0) = x_0, \end{cases} \tag{13}$$

where t is a fictitious time, $x_t = \frac{\mathrm{d}x}{\mathrm{d}t}$, $M : \Omega \times \mathbb{R} \to M_{N \times N}$ (where $M_{N \times N}$ denotes the set of matrix $N \times N$) and $d : \Omega \to \mathbb{R}^N$ is a function giving a descent direction. For example, assuming $h_0 \in C^1(\Omega, \mathbb{R})$, if $d = \nabla h_0$ and $M(x, t) = Id$ (the identity operator) for all $(x, t) \in \Omega \times \mathbb{R}$ we recover the steepest descent method.

According to previous notations, we use $V = \Omega$ and denote by $A_0(x_0) := A_0(x_0; t_0, \epsilon)$ the solution returned by the core optimization algorithm starting from the initial point $x_0 \in \Omega$ after $t_0 \in \mathbb{N}$ iterations and considering a stopping criterion defined by $\epsilon \in \mathbb{R}$. In this case, Problem (10) can be rewritten as:

$$\begin{cases} \text{Find } v \in \Omega \text{ such that} \\ \\ v \in \text{argmin}_{w \in \Omega} h_0(A_0(w)) \end{cases} \tag{14}$$

We consider a particular implementation of the algorithms A_i, $i = 1, ..., I$, introduced previously, to solve (14). For $i = 1, ..., I$, $A_i(v_1)$ is applied with a secant method (a low-cost method well adapted to find the zeros of a function (Mohammadi B. et al., 2002)) in order to perform the line search. It reads:

Step 1- Choose $v_2 \in \Omega$ randomly.

Step 2- For l from 1 to $t_{l_i} \in \mathbb{N}$ execute:

Step 2.1- If $h_i(v_l) = h_i(v_{l+1})$ go to **Step 3**

Step 2.2- Set v_{l+2} =proj$_\Omega(v_{l+1} - h_i(v_{l+1})\frac{v_{l+1}-v_l}{h_i(v_{l+1})-h_i(v_l)})$
where proj$_\Omega : \mathbb{R}^N \to \Omega$ is a projection algorithm over Ω defined by the user.

Step 3- Return the output: argmin$\{h_i(v_m), m = 1, ..., t_{l_i}\}$

This algorithm is denoted by **SDDA**.

3.2.3 SDA implementation with genetic core optimization algorithms

When a GA, described in Section 3.1, is used as the core optimization algorithm, problem (10) can be rewritten as:

$$\begin{cases} \text{Find } X^0 \in V = \Omega^{N_p} \text{ such that} \\ \\ X^0 \in \text{argmin}_{w \in \Omega^{N_p}} h_0(A_0(w)) \end{cases} \tag{15}$$

where $A_0(X^0) := A_0(X^0; N_p, N_g, p_m, p_c)$ with N_p, N_g, p_m, p_c parameters of the GA that here are considered fixed.

The solution of (15) may be determined, for instance, by using the SDA implementation presented in Section 3.2.1. However, a first numerical study (see (Ivorra , 2006) for more details) shows that the following variation of previous algorithms A_i (with $i = 1, ..., I$) is better adapted to the GA case. Let $X_1^0 = \{x_{1,j}^0 \in \Omega, j = 1, ..., N_p\}$. Then $A_i(X_1^0)$ reads:

Step 1- For l going from 1 to $t_{l_i} \in \mathbb{N}$ execute:

Step 1.1- Set o_l =argmin$\{h_0(x) : x \in A_{i-1}(X_l^0)\}$

Step 1.2- We construct $X_{l+1}^0 = \{x_{l+1,j}^0 \in \Omega, j = 1, ..., N_\text{p}\}$ as following:
$\forall j \in \{1, ..., N_\text{p}\}$, if $h_0(o_l) = h_0(x_{l,j}^0)$ set $x_{l+1,j}^0 = x_{l,j}^0$
else set $x_{l+1,j}^0 = \text{proj}_{\Omega^{N_P}}(x_{l,j}^0 - h_0(o_l)\frac{o_l - x_{l,j}^0}{h_0(o_l)-h_0(x_{l,j}^0)})$
where proj$_{\Omega^{N_P}} : \mathbb{R}^{N \times N_P} \to \Omega^{N_P}$ is a projection algorithm over Ω^{N_P} defined by the user.

Step 2- Return the output: argmin$\{h_i(X_m^0), m = 1, ..., t_{l_i}\}$

This version of the algorithm intends to optimize, individual by individual, the initial population of A_{i-1}. For each individual in the initial population:

- If there is a significant evolution of the cost function value between this individual and the best element found by A_{i-1}, the secant method used in Step 1.2 generates, in the optimized initial population, a new individual closer to this best element.
- If not, the secant method allows to create a new individual far from the current solution given by A_{i-1}.

Numerical experiments show that this coupling reduces the computational complexity of GAs (Ivorra , 2006; Ivorra B. et al., Accepted; Ivorra B. et al. in IJNME, 2006). In particular, this allows to consider smaller N_p and N_g numbers, compared with the case of applying GA alone. This algorithm is denoted by (**SDGA**).

4 Numerical test

4.1 Parameters in algorithms

We consider a CDMA binary codification of length $N_{\text{code}} = 8$ represented by the set of wavelengths $\Lambda = \{ \lambda_1 = 1.5465\mu m,\ \lambda_2 = 1.5473\mu m,\ \lambda_3 = 1.5481\mu m,\ \lambda_4 = 1.5489\mu m,\ \lambda_5 = 1.5497\mu m,\ \lambda_6 = 1.5505\mu m,\ \lambda_7 = 1.5513\mu m,\ \lambda_8 = 1.5521\mu m\}$.

The code c_1^A ='10110011' is characterized by $\Lambda_A^1 = [\lambda_1, \lambda_3, \lambda_4, \lambda_7, \lambda_8]$. We are interested to design a CDMA filter that reflects Λ_{A,λ_c}^1 with $\lambda_c = 1.5525\mu m$. This spectrum is depicted by Figure 3.

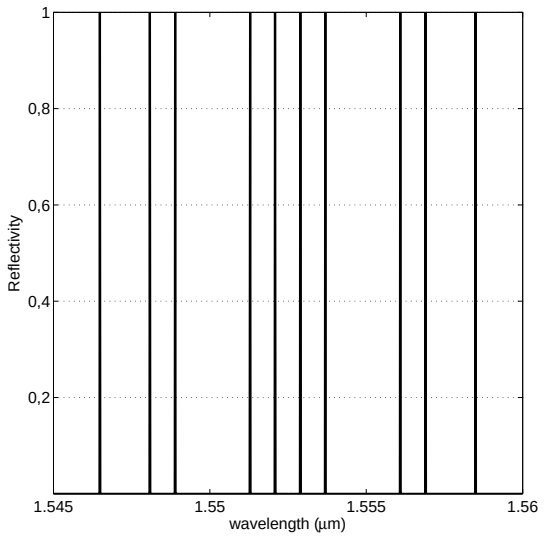

Figure 3: Target reflected spectrum presented in Section 4.1.

The SFBG characteristics are set to $n_{\text{eff}} = 1.45$, $L = 100$mm, $P = 1.039$mm and $\Theta = 0.53\mu$m. The SFBG apodization profiles are generated by $N_S = 9$ interpolation points with $\overline{n}_{\max} = 5.10^{-4}$ and the functional (7) is evaluated considering $N_c = 1200$ wavelengths in the transmission band $[1.545\mu\text{m}, 1.56\mu\text{m}]$. Those values have given good results (apodization profiles easy to implement) considering the problem of designing a multichannel optical filter of 16 peaks (Ivorra B. et al. in IJCSE, 2006).

In order to solve problem (8), considering previous values, we use the optimization methods presented in Section 3 (i.e. GA, SDDA and SDGA) with the following parameters:

- For SDDA: We use a two-layer algorithm (i.e. $I = 2$) with $t_0 = 10$, $t_{l_1} = 5$, $t_{l_2} = 5$ and $\epsilon = 0$. The initial point v_1 for A_2 is generated randomly in Ω_{N_S}. We consider $t_0 = 10$ iterations of the steepest descent algorithm (Mohammadi B. et al., 2002), which is is used as the core optimization algorithm A_0. The gradient of h_{0,N_c} used in A_0 is approximated considering a finite difference method.

- For SDGA: we use a one-layer algorithm (i.e. $I = 1$) with $t_{l_1} = 25$. The parameters considered for the GA, which is used as the core optimization algorithm, are the following:

  - The generation number and population size are set to $N_g = 10$ and $N_p = 10$, respectively.
  - The selection is a roulette wheel type (Goldberg D., 1989) proportional to the rank of the individuals in the population.
  - The crossover is barycentric in each coordinate with a probability of $p_c = 0.45$.

- The mutation process is non-uniform with a probability of $p_m = 0.35$.
- A one-elitism principle, that consists in keeping the current best individual in the next generation, has also been imposed.
- 10 iterations of the steepest descent method are performed at the end of the SDGA starting from the obtained solution.

- For GA: We use the same stochastic processes than SDGA but with a different set of parameters: $N_g = 30$, $N_p = 180$, $p_c = 0.35$, $p_m = 0.15$. 10 iterations of the steepest descent method are performed at the end of the GA starting from the obtained solution.

SDDA, SDGA and GA applied with those sets of parameters have been validated on various benchmark test cases (Ivorra , 2006; Ivorra B. et al., Submitted) and industrial applications (Ivorra B. et al. in IJNME, 2006; Hertzog D.E. et al., 2006; Debiane L. et al., 2006; Isebe D. et al., 2008), in particular on the design of pass-band and multichannel optical filters (Ivorra B. et al. in IJCSE, 2006; Ivorra , 2006).

4.2 Results and discussion

Figure 4 shows the apodization profiles obtained with SDDA, SDGA and GA and their associated reflected spectra. The convergence histories of each optimization process are presented in Figure 5. Results reported in this Section are summarized in Table 1.

For SDDA, the initial and final cost function h_{0,N_c} are equal to 12.84 and 2.09, respectively. The total number of functional evaluations is about 3000. SDDA optimization takes approximatively 10 hours real time in a 3.4GHz PC computer with 1 Gb Memory.

For SDGA, the final cost function h_{0,N_c} is equal to 2.31. The total number of functional evaluations is about 2700. SDGA optimization takes 9 hours.

For GA, the final cost function h_{0,N_c} is equal to 2.38. The total number of functional evaluations is about 5600. GA optimization takes 18 hours 40 minutes.

The three optimized apodization profiles have different shapes and are situated in distinct attraction basins of the function h_{0,N_c}. This points out the fact that h_{0,N_c} is highly non convex and the optimization problem (8) difficult to solve. This is confirmed by observing the convergence history of the SDDA, which shows that, during the optimization process, the steepest descent algorithm has visited various attraction basins and found different local minima.

From a numerical points of view, both SDDA and SDGA have found better results than GA and are less time consuming.

From an implementation point of view, all optimized apodization profiles present interesting characteristics:

- As we can observe on Figure 3, the reflected spectra associated to the optimized profiles correspond to good approximations of the target reflected spectrum.

- The optimized apodization profiles (see Figure 4) are suitable for practical implementation. Indeed, the number of necessary π-phase shifts is 5 (a number easy to implement), the index modulation of the profile is uniformly distributed along the pattern and the maximum index variation of the profile is inferior to 3.10^{-4}, which is a reasonable level (Ivorra B. et al. in IJCSE, 2006).

- A stability analysis on the reflected spectra, when applying a random perturbation of 10% on the optimized apodization profiles, show that all optimized results have a small variation of $\approx$ 4.3% on their reflected spectrum. This is important because, due to technical limitations, small perturbations could appear in the apodization profile during the writing process.

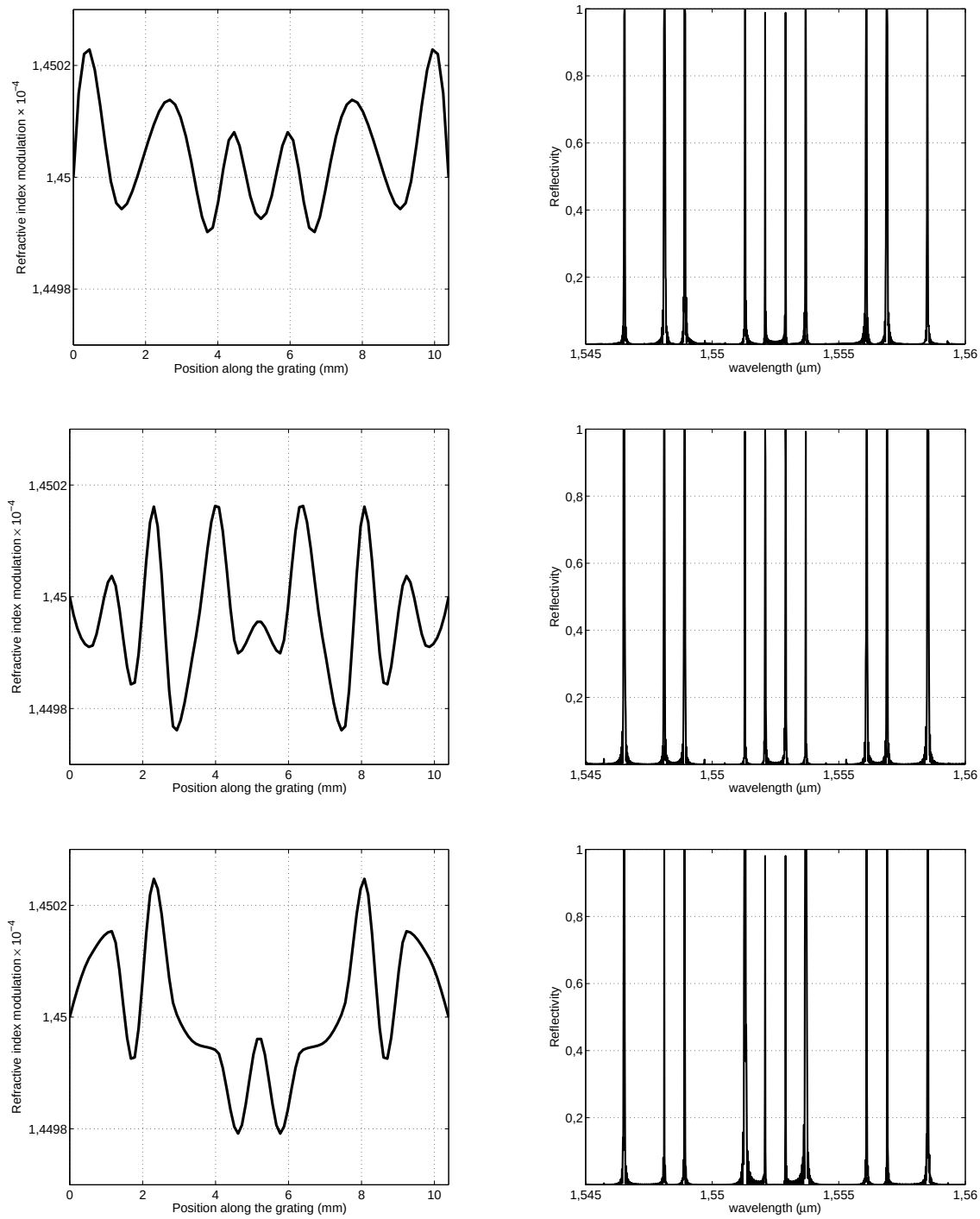

Figure 4: (**Left**) Optimized apodization profiles and (**Right**) associated reflected spectrum obtained by SDDA (**Up**), SDGA (**Middle**) and GA (**Bottom**) optimization methods.

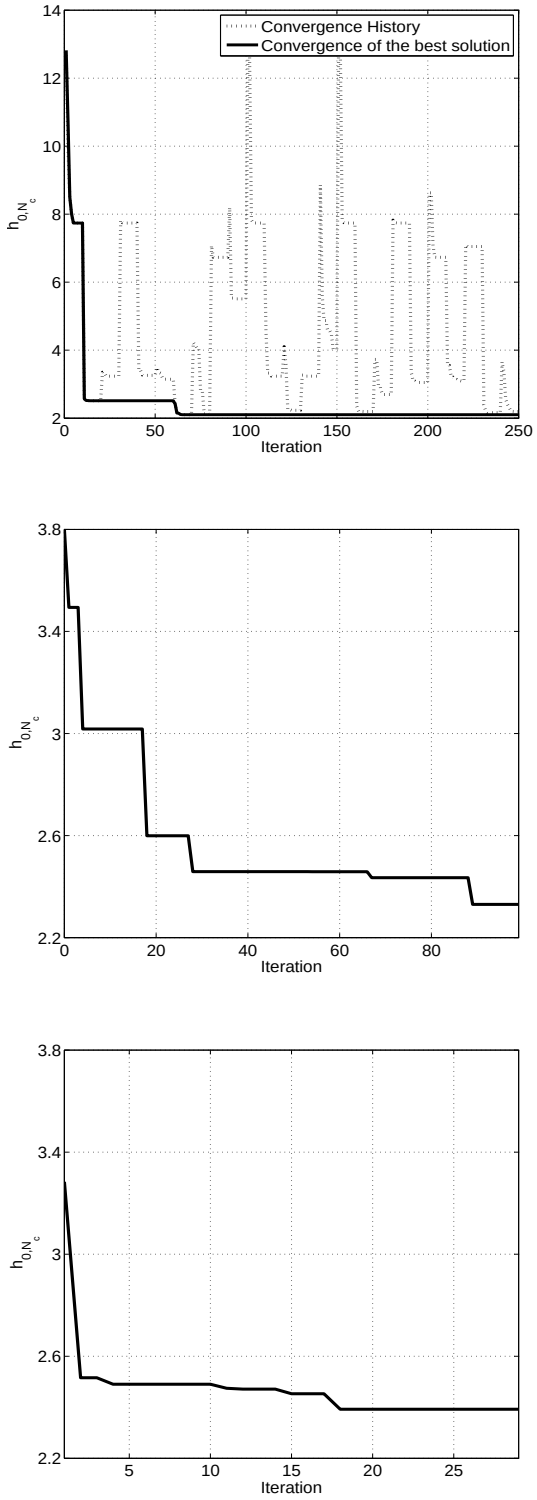

Figure 5: Best element convergence history vs. iterations (—) and global convergence history vs. iterations (**...**) for SDDA (**Up**). Best element convergence history vs. iterations for SDGA (**Middle**) and GA (**Bottom**).

Optimization Method	Cost Function Final Value	Evaluation Number
SDDA	2.09	3000
SDGA	2.31	2700
GA	2.37	5600

Table 1: Numerical results obtained considering SDDA, SDGA and GA optimization methods , (**Center**) value of h_{0,N_c} of the best element found by the optimization algorithm, (**Right**) Number of evaluation of the function h_{0,N_c} needed by the optimization algorithm.

5 Conclusion

A particular code division multiple access filter based on sampled fiber Bragg grating has been designed using three particular optimization algorithms: two original semi-deterministic algorithms (SDDA and SDGA) and a genetic algorithm (GA). The apodization profiles produced by those optimization approaches exhibit good characteristics for practical implementation because they have no steep variation, a low maximum index modulation values and small numbers of π-phase shifts. Also, their associated reflected spectrum are weakly sensitive to perturbations. However, SDDA and SDGA have produced better solutions and need less computational time than GA alone.

A next step, could be the study of the effect of combined apodization and phase profiles optimization (Rothenberg J.E. et al., 2002) in order to avoid the symmetry in spectra mentioned previously. During this work, we have been interested only by apodization optimization to keep a grating easy to implement by any optical laboratory. Indeed, phase variation requires more complex and expansive materials.

A Matlab© version of the algorithms presented in this paper are included in the free optimization package *Global Optimization Platform (GOP)* which can be downloaded at:

http://www.mat.ucm.es/momat/software.htm.

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